CBCS Scheme

USN | | | | | | | | | | |

15MATDIP31

Third Semester B.E. Degree Examination, June/July 2018
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the modulus and amplitude of $\frac{(1+i)^2}{3+i}$. **(05 Marks)**

b. Prove that $\left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n = \cos n\theta + i\sin n\theta$. **(05 Marks)**

c. If $z = \cos\theta + i\sin\theta$, then show that $x^n + \frac{1}{x^n} = 2\cos n\theta$, $x^n - \frac{1}{x^n} = 2i\sin n\theta$. **(06 Marks)**

OR

2 a. Find the sine of the angle between $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$. **(05 Marks)**

b. Find the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ **(05 Marks)**

c. Show that (3, –2, 4), (6, 3, 1), (5, 7, 3) and (2, 2, 6) are coplanar. **(06 Marks)**

Module-2

3 a. Find the n^{th} derivative of $\sin(3x)\cos x$. **(05 Marks)**

b. Find the angle between radius vector and tangent to the curve $\gamma^m \cos m\theta = a^m$. **(05 Marks)**

c. Find the pedal equation of $\gamma = a(1 + \cos\theta)$. **(06 Marks)**

OR

4 a. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin(2u)$. **(05 Marks)**

b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(05 Marks)**

c. If $u = x + y$, $v = y + z$, $w = z + x$, find $J\left(\frac{uvw}{xyz}\right)$. **(06 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi} x\cos^6 x\,dx$. **(05 Marks)**

b. Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^6)^{7/2}}\,dx$ **(05 Marks)**

c. Evaluate $\int_0^1 x^5(1-x^2)^{5/2}\,dx$. **(06 Marks)**

OR

6 a. Evaluate $\int_1^2\int_3^4 (xy+e^y)\,dy\,dx$. **(05 Marks)**

b. Evaluate $\int_0^1\int_x^{\sqrt{x}} xy\,dy\,dx$. **(05 Marks)**

c. Evaluate $\int_0^1\int_0^1\int_0^y xyz\,dx\,dy\,dz$. **(06 Marks)**

Module-4

7 a. Find the angle between the tangents to the curve $x=t^2, y=t^3, z=t^4$ at $t=2$, and $t=3$. **(05 Marks)**

b. Find the unit normal to the curve $\vec{\gamma} = 4\sin t\,\hat{i} + 4\cos t\,\hat{j} + 3t\,\hat{k}$. **(05 Marks)**

c. Find the velocity and acceleration to the curve $\vec{\gamma} = t^2\,\hat{i} - t^3\,\hat{j} + t^4\,\hat{k}$ at $t=1$. **(06 Marks)**

OR

8 a. Find the directional derivative of $\varphi = x^3y^3z^3$ at (1, 2, 1) in the direction of $\hat{i}+2\hat{j}+2\hat{k}$. **(05 Marks)**

b. Find the unit normal to the surface $xy + x + zx = 3$ at $(1,1,1)$. **(05 Marks)**

c. If $\vec{F} = \nabla(x^3+y^3+z^3-3xyz)$, find div $\vec{F}$. **(06 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$. **(05 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \sin x$. **(05 Marks)**

c. Solve $y(x + y)dx + (x + 2y - 1)dy = 0$. **(06 Marks)**

OR

10 a. Solve $(x^2 + y)dx + (y^3 + x)dy = 0$. **(05 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$. **(05 Marks)**

c. Solve $(x^2 + y^2)\frac{dy}{dx} = xy$. **(06 Marks)**

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